Quantum computing using continuous-time evolution

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GOAL: increase computing power . . .

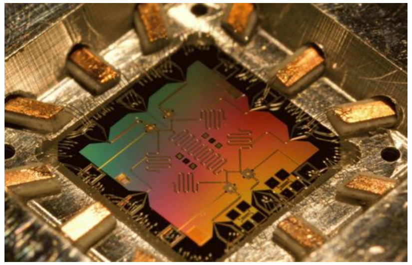
★ *current computers already very powerful*
– two barriers to more computing power:

1. silicon chip technology reaching limits
2. energy consumption far from optimal:
   – *resource limits; global warming*

[lots of room to improve on energy consumption
– see, e.g., SpiNNaker project for other ways to use Si]

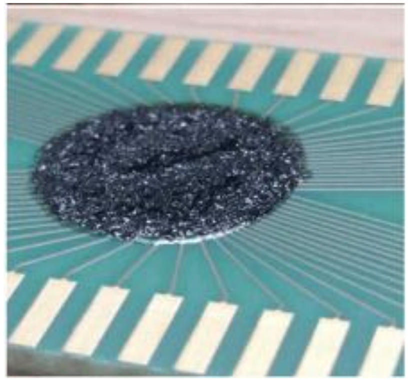
note these are related: can't cool Si chips any faster

beyond silicon . . .

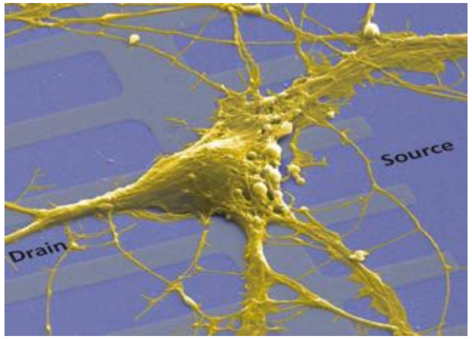


quantum: IBM 5 qubit

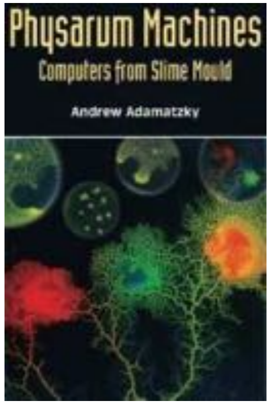
BZ reaction chemical

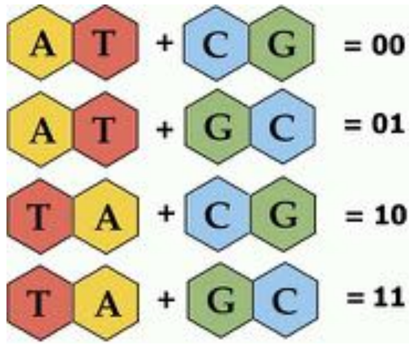


reservoir computer



rat neuron on silicon





encoding for DNA computer

★ future computing is diversifying ★

⇒ *need to co-design algorithms with hardware* ⇐

hybrid computers . . .

practice: co-processors:

conventional:

- graphics cards
- ASIC application-specific integrated circuit
- FPGA field-programmable gate array

unconventional: control + substrate:

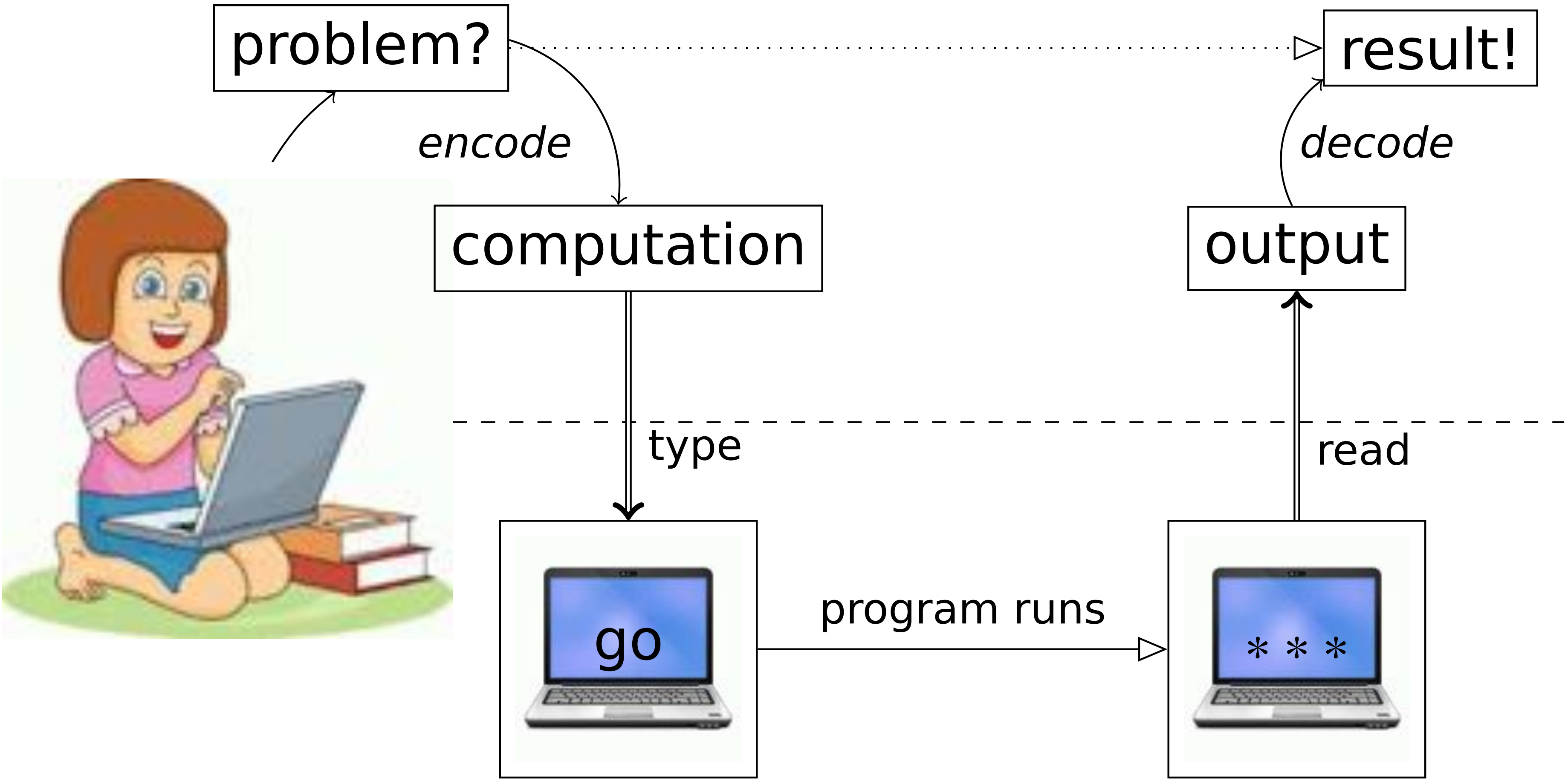
- quantum
- NMR
- reservoir
- slime mould

★ hybrid computational systems are the norm ★

theory: single paradigm:

- classical – Turing Machine
- analog – Shannon's GPAC
- quantum – gate model, QTM, CV . . .
- linear optics (Bosons) `[Aaronson/Arkhipov STOC 2011 ECCC TRI-10 170]`

computing



article: “When does a physical system compute? Proc. Roy. Soc. A 2014 **470**, 20140182

http://dx.doi.org/10.1098/rspa.2014.0182 (Horsman/Stepney/Wagner/VK)

quantum computing

input $\longrightarrow$ $\boxed{\text{encode}}$ $\longrightarrow |\psi_{in}\rangle \longrightarrow \hat{U} \longrightarrow |\psi_{out}\rangle \longrightarrow$ $\boxed{\text{decode}}$ $\longrightarrow$ result

$\hat{U}$ is unitary evolution (or more generally, open system/environment)
– can be gate sequence, or engineer Hamiltonian $\hat{H}(t)$ such that

$$|\psi_{out}\rangle = \mathcal{T}\exp\{-i/\hbar\int dt\ \hat{H}(t)\}\,|\psi_{in}\rangle$$

★ covers most of quantum information processing . . .
. . . including communications, where aim is *result=input*

$\boxed{\text{encode}}$ – arbitrary choices:

using spin-down $|\downarrow\rangle \equiv 0$ instead of spin-up $|\uparrow\rangle \equiv 0$ makes no difference
$\rightarrow$ provided $\boxed{\text{encode}}$ and $\boxed{\text{decode}}$ done consistently

quantum information processing

Quantum Information is built on the idea that:

Quantum Logic allows greater efficiency than **Classical Logic**

classical	**quantum**
bits, 0 or 1	qubits, $\alpha\|0\rangle + \beta\|1\rangle$
yes or no, binary decisions	yes and no, superpositions
HEADS or TAILS, random numbers	random measurement outcomes

$\Rightarrow$ quantum gives different computation from classical: how different?

- **computability** – what can be computed?
- **complexity** – how efficiently can it be computed?

$\Rightarrow$ quantum **computability** is the same as classical
complexity differs: some problems can be computed more **EFFICIENTLY**

encoding matters . . .

. . . it determines the physical resources required:

Number	*Unary*	*Binary*
0		0
1	•	1
2	••	10
3	• • •	11
4	• • ••	100
...	...	...
2x4	• • ••	1000
=8	• • ••	
...	...	...
N	$N \times \bullet$	$\log_2 N$ bits

Read out:

Unary: measurements with N outcomes

Binary: $\log_2 N$ measurements with 2 outcomes each

$\longrightarrow$ exponentially better for precision

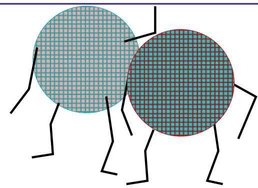
[Ekert & Jozsa PTRSA 356 1769-82 (1998)]

$\longrightarrow$ exponential reduction in memory

[does not have to be binary: Blume-Kohout, Caves, I. Deutsch, Found. Phys. 32 1641-1670 (2002)]

★ floating point: 0.1234567×10^{89} even more efficient, trade precision/memory ★

gate model quantum computing



the standard introduction . . .

qubits: 2-state quantum systems: *examples:* electron spin, photon polarisation

localised: distinguishable – no Fermi or Bose statistics to worry about

choose a basis: $|0\rangle$ and $|1\rangle$ from which superpositions $\alpha\,|0\rangle + \beta\,|1\rangle$

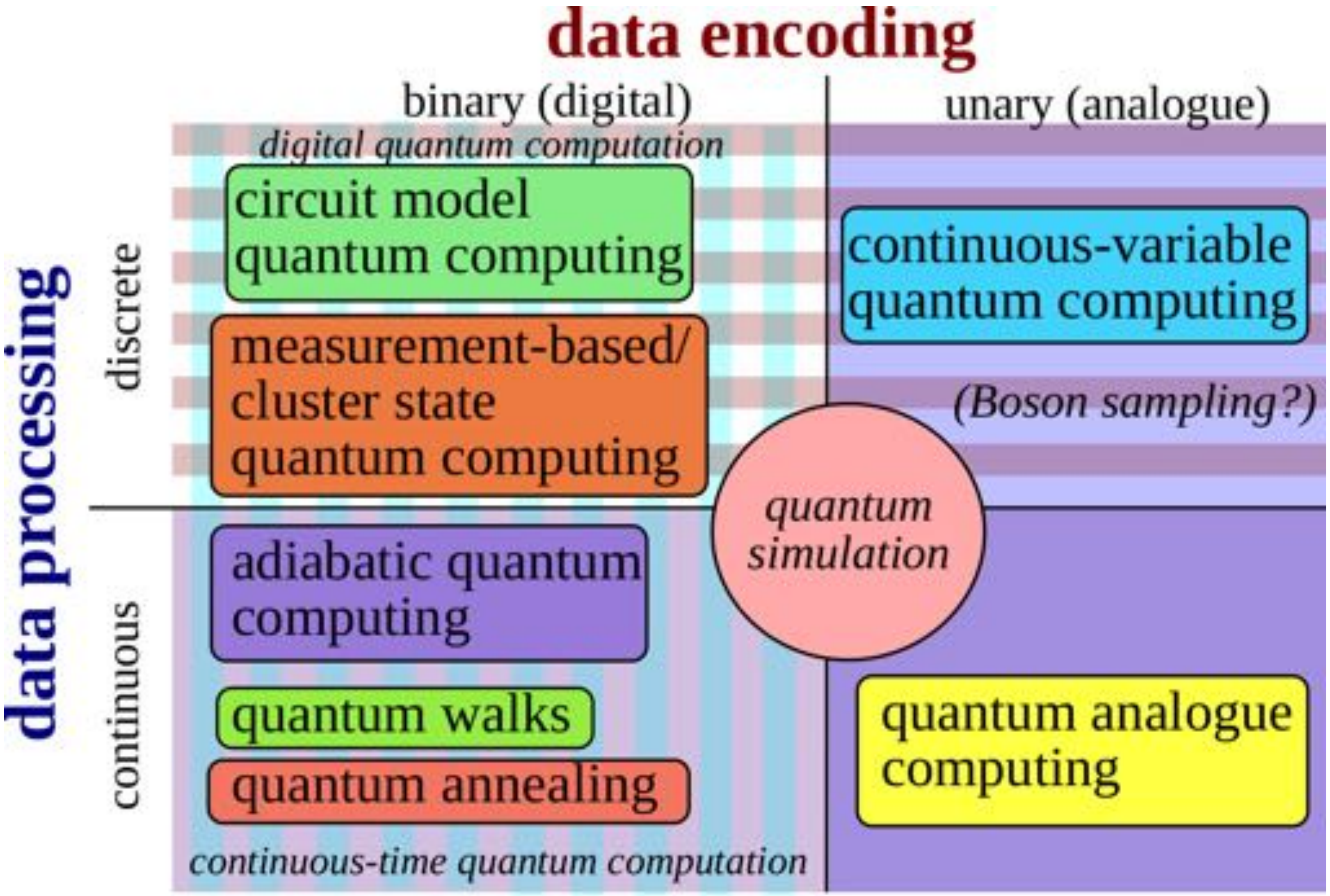
apply gates: universal sets, e.g., Hadamard + CNOT + T

add error correction . . .

BUT . . .

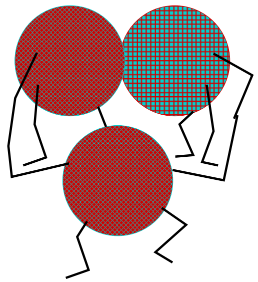
why do we expect quantum computing to be like digital classical computing?

. . . when we have a diverse range of models under development?



qubits vs bits

bits are either zero or one – *flip between the two values*



qubits can be any superposition: $\alpha\,|0\rangle + \beta\,|1\rangle$

– *can change smoothly from zero to one or anything in between*

★ **hence** ★

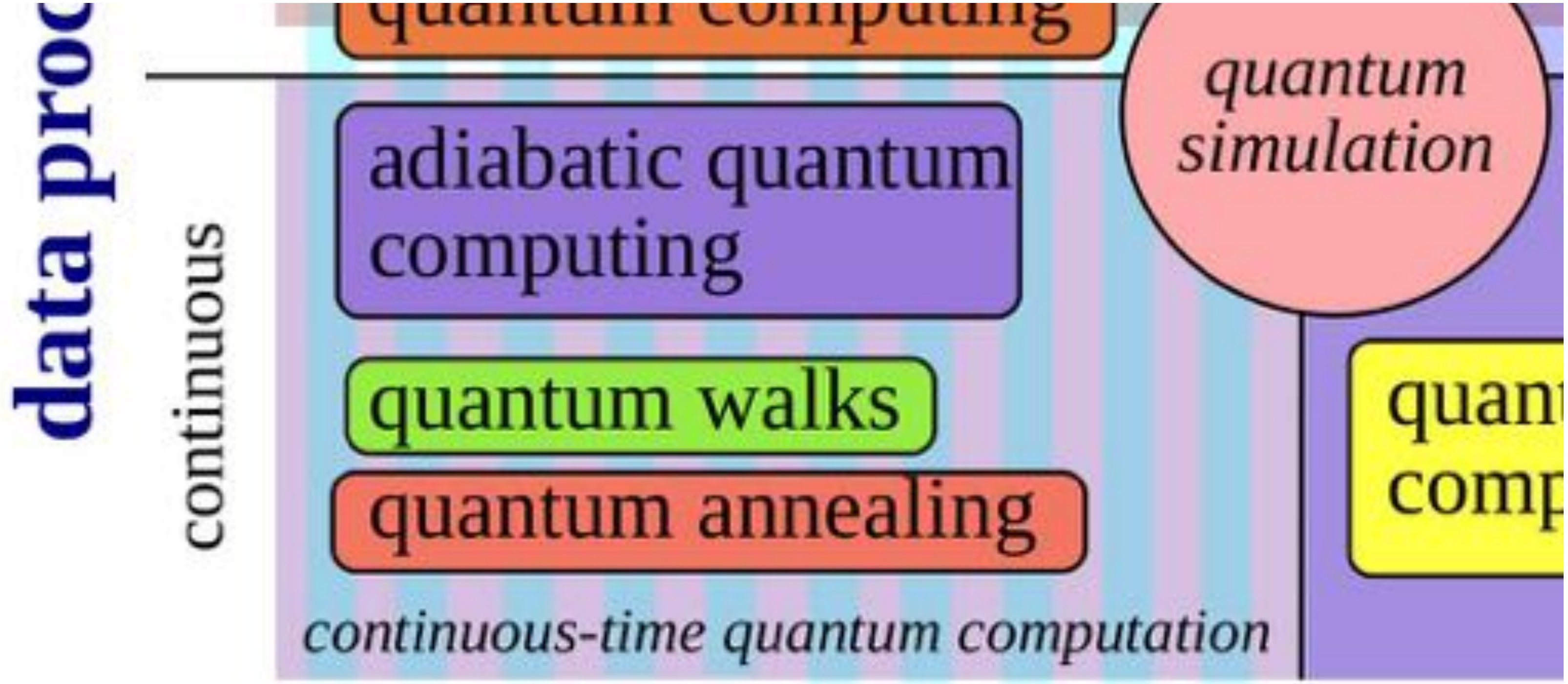
discrete gates make sense for **bits**: *bit-flip is all you can do*

for **qubits**: exact bit-flip is just as hard as other rotations

continuous-time evolution makes sense for **qubits**

+ this is related to work on foundations by Lucien Hardy (2001)
"Quantum theory from five reasonable axioms." arχiv:quant-ph/0101012

continuous-time quantum computing



⟶ take a closer look at what is in the continuous-time corner . . .

encoding problems into qubit Hamiltonians

+ computational basis state $|j\rangle = |q_0 q_1 \ldots q_k \ldots q_{n-1}\rangle$ with $q_k \in \{0,1\}$
+ superposition of all basis states:

$$|\psi_0\rangle = 2^{-n/2} \sum_{j=0}^{2^n-1} |j\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)^{\otimes n}$$

encode problem into n-qubit Hamiltonian $\hat{H}_p$

such that solution is lowest energy state (ground state)

example: find state $|m\rangle$ then $\hat{H}_p = \mathbb{1} - |m\rangle\langle m|$

$$\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

example: three qubits, exactly one must be $|1\rangle$

$$\hat{H}_p = (\mathbb{1} - \hat{Z}_1 - \hat{Z}_2 - \hat{Z}_3)^2$$

$$\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pauli-Z operator: $\hat{Z}\,|0\rangle = |0\rangle$ and $\hat{Z}\,|1\rangle = -\,|1\rangle$

adiabatic quantum computing

given $\hat{H}_p$ [Farhi et al, quant-ph/0001106]

initialise in ground state $|\psi_{\text{init}}\rangle$ of simpler Hamiltonian $\hat{H}_0$ – easy to prepare –

transform adiabatically:

$$\hat{H}(t) = [1 - s(t)]\hat{H}_0 + s(t)\hat{H}_p$$

with *annealing parameter* $s(t=0) = 0$ and $s(t = t_f) = 1$

$s(t)$ monotonically increasing, function of size of problem space $N = 2^n$ and the *accuracy parameter* ϵ determined by adiabatic condition,

$$\frac{\left|\left\langle \frac{d\hat{H}}{dt} \right\rangle_{1,0}\right|}{(E_1 - E_0)^2} \equiv \epsilon \ll 1, \tag{1}$$

0 and 1 refer to the ground and excited states, and $\left|\left\langle \frac{d\hat{H}}{dt} \right\rangle_{1,0}\right| \equiv \langle E_1| \frac{d\hat{H}}{dt} |E_0\rangle$

closer ϵ is to zero – the more completely the system will stay in the ground state and the longer the computation will take

encoded hypercubes for quantum walks

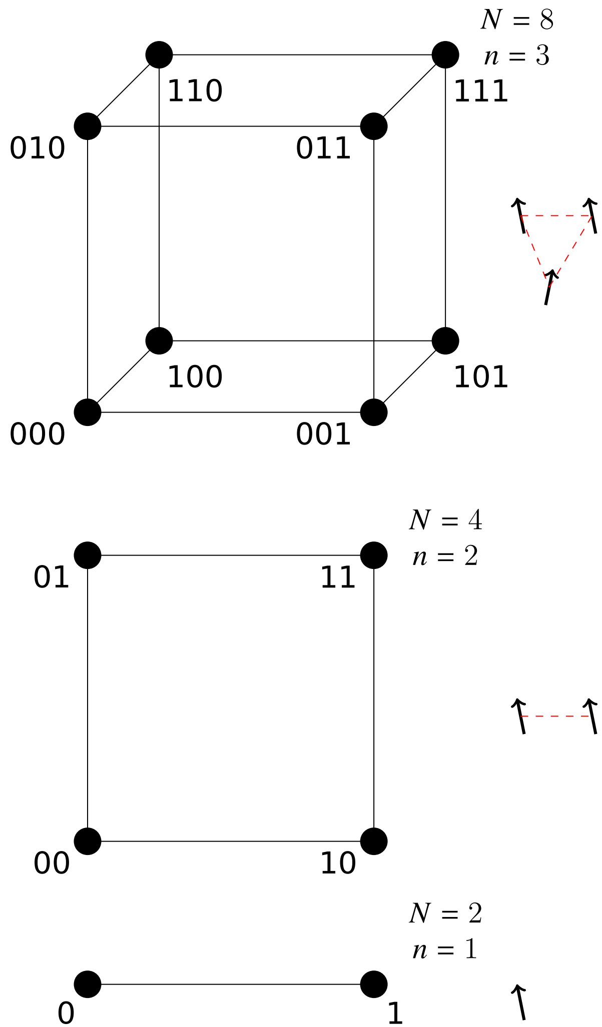
n qubits encode 2^n vertices:

for a hypercube graph, $\hat{H}_h = \gamma\left(n\mathbb{1} - \sum_j \hat{X}_j\right)$

where j is the qubit label: $j = 0 \ldots n-1$

Pauli-X operator $\hat{X}_j$
bit-flips qubit j $\quad 0 \leftrightarrow 1$

$\longrightarrow$ this moves the position of the quantum walker along an edge of the hypercube



continuous-time quantum search

find the marked state:
the problem Hamiltonian

$$\hat{H}_p = \hat{H}_m = \mathbb{1} - |m\rangle\langle m|$$

– makes $|m\rangle$ lower energy –
★ use the hypercube Hamiltonian $\hat{H}_h$ for the easy Hamiltonian/initial state
– ground state is superposition over all states $|\psi(t=0)\rangle = \{(|0\rangle + |1\rangle)/\sqrt{2}\}^{\otimes n}$

in Pauli operators:

$$\hat{H}_m = \mathbb{1} - \frac{1}{2^n}\prod_{j=1}^{n}(\mathbb{1} + q_j\hat{Z}_j),$$

where $q_j \in \{-1, 1\}$ defines bitstring corresponding to m for $-1 \equiv 0$ to convert to bits; for gadgets to implement this:

Dodds/VK/Adams/Chancellor arχiv:1812.07885

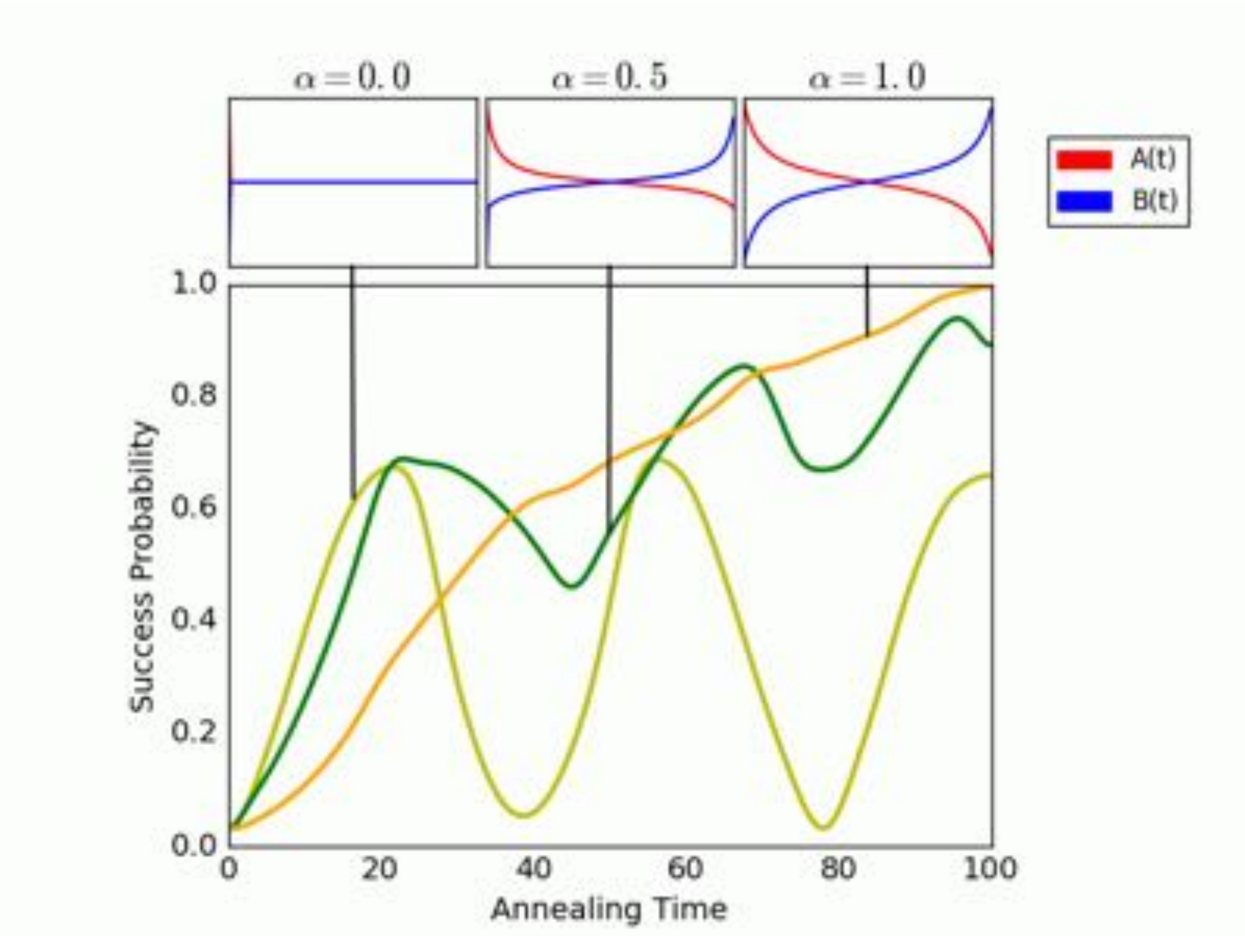
$$\hat{H}(t) = A(t)\hat{H}_h + B(t)\hat{H}_m$$

apply time-evolution

$$|\psi(t_f)\rangle = \mathcal{T}\exp\{-i\int dt\ \hat{H}(t)\}\,|\psi(t=0)\rangle$$

measure after suitable time $t_f \propto \sqrt{N}$ to obtain quantum speed up

hybrid continuous-time quantum search algorithms



interpolate between QW ($\alpha = 0$) and AQC ($\alpha = 1$)

$$\hat{H}(\alpha,t) = A(\alpha,t)\hat{H}_h + B(\alpha,t)\hat{H}_m$$

$$\hat{H}_{QW} = \gamma\hat{H}_h + \hat{H}_m$$

$$\hat{H}_{AQC} = [1 - s(t)]\hat{H}_h + s(t)\hat{H}_m$$

$\rightarrow$ need γ and $s(t)$ . . .

[James Morley's work (UCL CDT) PRA 99, 022339 (2019) arχiv:1709.00371]

more realistic problems

Sherrington Kirkpatrick spin glasses: frustrated spin systems

★ NP-hard for finding ground state *i.e., expect polynomial speed up*

★ more like realistic hard optimisation problems

$$\hat{H}_p = -\sum_{j=0}^{n=1}\sum_{k=j+1}^{n-1}\frac{J_{jk}+J_{kj}}{2}\hat{Z}_j\hat{Z}_k - \sum_{j=0}^{n=1}h_j\hat{Z}_j$$

J_{jk}, h_j drawn from Gaussian distributions with mean $= 0$ (hardest)

- AQC can find ground states faster than guessing

  [e.g., Martin-Mayor/Hen Sci Rep 5, 15324 (2015); arXiv:1502.02494]

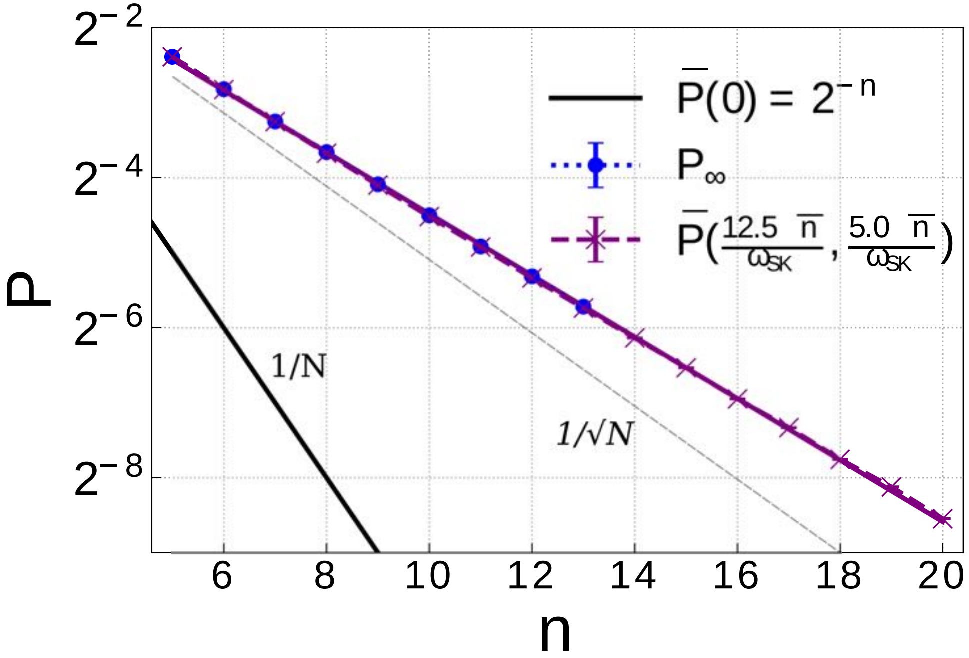
$$\hat{H}(t) = (1 - s(t))\hat{H}_w + s(t)\hat{H}_p$$

- *what about continuous-time quantum walks?*

$$\hat{H}(t) = \gamma\hat{H}_w + \hat{H}_p$$

- compare with a random energy model (REM)

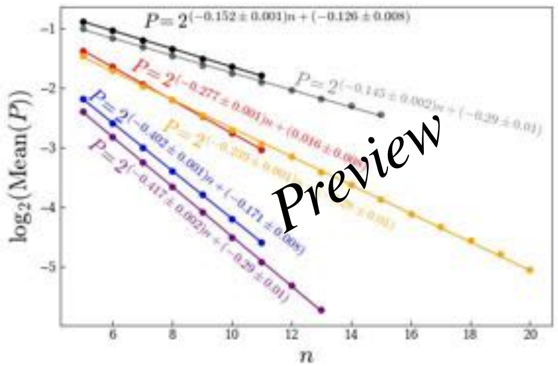
SK spin glass results



success probability scaling

for heuristic γ based on energy scales

(average of 10,000 random instances each data point)



hybrid algorithms do even better...

don't need to solve problem to set parameters, heuristic does well

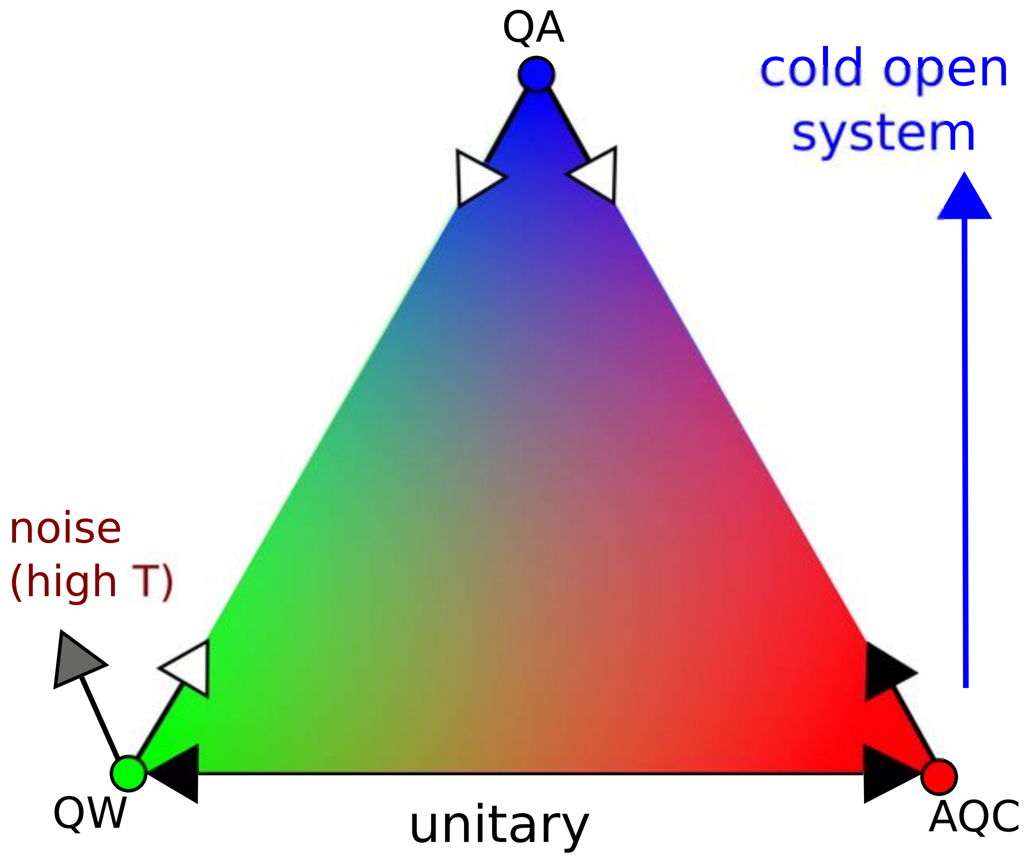
P $\sim N^{-0.41}$ for short run times [Callison/Chancellor/Mintert/VK arχiv:1903.05003]

cf P $\sim N^{-0.5}$ for search, i.e., ★ better than search ★

continuous-time quantum computing

family of computational models:

- **discrete** – qubits for efficient encoding
- **continuous time** evolution with engineered Hamiltonian
- coupling to low temp bath – open system effects **cooling**



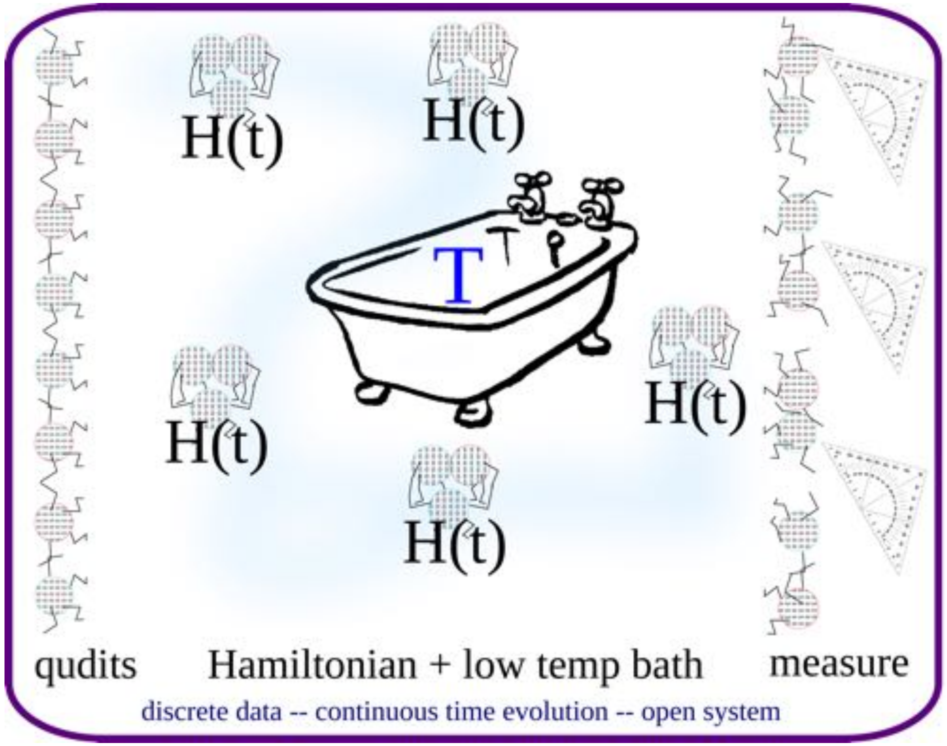
$\longrightarrow$ makes sense because **qubits** do superposition; classical **bits** don't

exploits natural properties of quantum systems

quantum annealing

– a noisy version of adiabatic quantum computing – run faster than adiabatic –

low temperature bath helps remove energy to reach ground state solution . . .



. . . **but** may also **kill helpful coherences** in the process

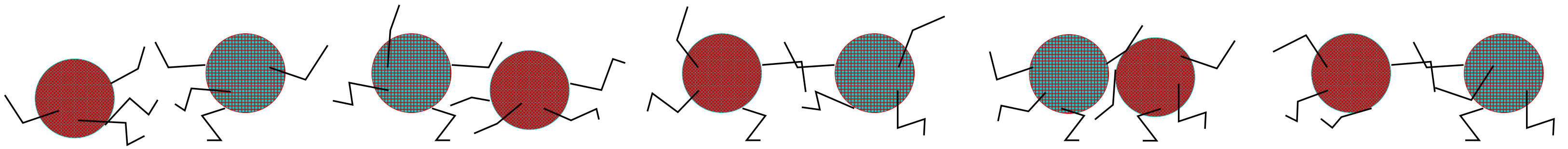
[in literature, AQC and Qanneal used interchangeably → confusion]

scaling and limitations

scaling: more qubits requires more precise Hamiltonian parameters
– *evidence from D-Wave that they are reaching that limit at 2000 qubits*

- this is still way more than classical computers can simulate
- limit could be higher for other hardware
- use as quantum co-processors for bottleneck subroutines
- hybrid algorithms, e.g., Chancellor NJP 19, 2, 023024 (2017) & arχiv:1609.05875

error correction: standard digital methods don't apply;
+ quantum codes can provide robustness (e.g., Lidar PRL 100 160506 2008)
+ NMR and other quantum control techniques also help



UK Quantum Computing landscape

UK NATIONAL QUANTUM TECHNOLOGIES PROGRAMME

- 2014: £270M over five years, four Hubs:
  imaging, metrology, communications, **computing**

- 2019: £80M Hub renewal + £200M+ innovation funding
  + National Quantum Computing Centre (NQCC) ($\sim$ £80M)

Global context:

- EU Flagship on Quantum Technology + big national investments:
  ★ partnerships with foundaries (e.g., Intel/Delft)
  → US: NSF already funded testbeds like NQCC aims to develop

- major companies investing in hardware+software
  (IBM, Google, Microsoft ...)

- many new hardware and software start ups . . .